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Kirkegaard, Poul Henning

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FRACTURE AND DYNAMICS
PAPER NO. 39

To be presented at the "11th International Modal Analysis Conference and Exhibition", Kissimmee, Florida, USA, February 1-4, 1993

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P. H. Kirkegaard

Assistant Professor of Civil Engineering
Department of Building Technology and Structural Engineering
Aalborg University
Sohngaardsholmsvej 57, 9000 Aalborg, Denmark

ABSTRACT

Optimal selection of the sampling interval for estimation of the modal parameters by an ARMA-model for a white noise loaded structure modelled as a single degree-of-freedom linear mechanical system is considered. An analytical solution for an optimal uniform sampling interval, which is optimal in a Fisherian sense, is given. The solution is investigated by a simulation study. It is shown that if the experimental length T_f is fixed it may be useful to sample the record at a high sampling rate, since more measurements from the system are then collected. No optimal sampling interval exists. But if the total number of sample points N is fixed an optimal sampling interval exists. Then it is far worse to use a too large sampling interval than a too small one since the information losses increase rapidly when the sampling interval increases from the optimal value.

NOMENCLATURE

- T_f experimental length
- N number of sample points
- $Y(t)$ stochastic response process
- $y(t)$ realization of $Y(t)$
- ζ damping ratio
- f eigenfrequency
- Δt sampling interval
- $U(t)$ white noise excitation process
- ω_n natural frequency
- ω_d damped natural frequency
- Φ_i Auto Regressive parameters
- $\mathcal{O}_1$ Moving Average parameters
- $\mathcal{E}(t)$ noise process
- $e(t)$ realization of $\mathcal{E}(t)$
- $\overline{\overline{J}}$ Fisher information matrix
- $\lambda_\mathcal{E}$ variance of noise process
- $E[\cdot]$ expectation operator
- q shift operator
- J_{ij} elements in Fisher information matrix
- P variable in equation (5)
- $p1$ variable in equation (6)
- $p2$ variable in equation (6)
- z complex number
- $\overline{\Phi}$ vector including AR-parameters
- $\overline{\Psi}(t, \overline{\Phi})$ stochastic process given by equation (8)
- $\overline{\psi}(t, \overline{\Phi})$ realization of $\overline{\Psi}(t, \overline{\Phi})$
- $\epsilon(t, \overline{\Phi})$ stochastic process given by equation (9)
- $\overline{\overline{B}}$ matrix in equation (34)
- $\overline{\overline{A}}$ transformation matrix given by $\overline{\overline{B}}$
- $\overline{\theta}$ modal parameter vector
- $\hat{\overline{\theta}}_N$ estimate of modal parameter vector
- $\mu_{\hat{\overline{\theta}}_N}$ expected value of $\hat{\overline{\theta}}_N$
- $\overline{\overline{C}}_{\hat{\overline{\theta}}_N}$ covariance matrix
- N_{sim} number of simulations
- det determinant
- tr trace
- δf coefficient of variation of f
- $\delta\zeta$ coefficient of variation of ζ
- Δt^{opt} optimal sampling interval

1. INTRODUCTION

Design of experiment in dynamic system identification is the problem of choosing the experimental conditions (test signals, sampling strategy, location of sensors etc.) so that the information provided by the experiment is maximized. Optimal design of a system identification experiment is usually obtained by minimizing a scalar measure, e.g. the determinant, the trace etc., of an estimated parameter covariance matrix, based on prior knowledge , see e.g. Goodwin et al. [1]. The covariance matrix is normally estimated by the inverse of the Fisher information matrix implying design of experiments which are optimal in a Fisherian sense. Design of experiments for parametric identification of dynamic systems has been a subject of research during the last decades mainly developed in relation to identification of electrical systems, see e.g. Goodwin et al. [1], Gevers et al. [2], Wahlberg et al. [3] , Payne et al. [4], Ljung [5] and Söderström et al. [6]. Design of experiments in relation to structural problems seems to be a subject which only has received little attention during the last decade and will be a subject of research in the future, see e.g. Kirkegaard [7].

The aim of this paper is to investigate how the sampling interval can be optimally selected for estimation of the modal parameters by an ARMA model for ambient test of a civil engineering structure. It is unavoidable that sampling as such leads to information losses and it is important to select the sampling interval, so that these losses are insignificant. It is well-known from the Shannon sampling theorem, see e.g. Bendat et al. [8], that the only frequencies which may be reconstructed from a sampled continuous signal, sampled at equally spaced intervals Δt, are those up to half the sample frequency, the so-called Nyquist frequency. Further, the intuition says that the higher sampling rate the better a discrete representation of the continuously measured signal will be obtained. But, in this paper it will be shown that this is not true in general.

In section 2 an analytical expression for the covariance matrix of the modal parameters estimated by an ARMA-model is established. It will be shown that an analytical relation between sampling interval and the covariance matrix can be obtained. Next in section 3 this analytical expression is investigated by a simulation study. At last in section 4 and 5, respectively, conclusions and references are given.

2. THEORY

A single degree-of-freedom mechanical vibrating system is considered where the stochastic response process $\{Y(t)\}$ is the solution to the second order differential equation

$$\ddot{Y}(t) + 2\zeta(2\pi f)\dot{Y}(t) + (2\pi f)^2 Y(t) = U(t) \qquad (1)$$

f is the eigenfrequency implying that the natural undamped frequency $\omega_n = 2\pi f$. ζ is the damping ratio and $\{U(t)\}$ is stationary zero mean Gaussian white noise process.

A proper discrete model for the second order SDOF continuous system excited by white noise is an ARMA(2,1) model. This discrete model is given by, see e.g. Pandit et al. [9]

$$y(t) = \Phi_1 y(t-1) + \Phi_2 y(t-2) + e(t) - \mathcal{O}_1 e(t-1) \qquad (2)$$

where the discrete time, the sampling interval, is Δt. Φ_1, Φ_2 is the Auto Regressive (AR) parameters, $\mathcal{O}_1$ is the Moving Average (MA) parameter and $e(t)$ is a time series of independent Gaussian distributed numbers. The ARMA parameters are given by, see e.g. Pandit et al. [9]

$$\Phi_1 = 2e^{-\zeta\omega_n\Delta t}\cos(\omega_d\Delta t) \qquad (3)$$

$$\Phi_2 = -e^{-2\zeta\omega_n\Delta t} \qquad (4)$$

$$\mathcal{O}_1 = -P \pm \sqrt{P^2 - 1} \qquad (5)$$

where

$$P = \frac{\omega_n \sinh(2p_1) - \zeta\omega_n \sin(2p_2)}{2\zeta\omega_n \sin(p_2)\cosh(p_1) - 2\omega_d \sinh(p_1)\cos(p_2)} \qquad (6)$$

$p_1 = \zeta\omega_n\Delta t$ and $p_2 = \omega_d\Delta t$. $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency.

The problem is now whether there is an optimal sampling rate or not if the parameters $\overline{\theta} = [f, \zeta]^T$ should be estimated from an experiment.

From measurements of the response process it is possible to get unbiased estimates of the AR-parameters Φ_1 and Φ_2, see e.g. Pandit et al. [9], where estimates of the variances of the estimated parameters can be estimated by the Cramer-Rao lower bound. This implies that the covariance matrix of parameter estimates can be obtained by the inverse of the Fisher information matrix $\overline{\overline{J}}$ which can be written

$$\overline{\overline{J}} = \frac{N}{\lambda_{\mathcal{E}}} E[\overline{\Psi}(t,\overline{\Phi})^T \overline{\Psi}(t,\overline{\Phi})] \qquad (7)$$

A realization of the stochastic process $\{\overline{\Psi}(t,\overline{\Phi})\}$is given by

$$\overline{\psi}(t,\overline{\Phi}) = \frac{\partial\epsilon(t,\overline{\Phi})}{\partial\overline{\Phi}} \qquad (8)$$

It is assumed that the variance of the noise process $\{\mathcal{E}(t)\}$ is $\lambda_{\mathcal{E}}$. N is the number of samples. $\overline{\Phi}$ is a vector including the AR-parameters

The prediction error $\epsilon(t,\overline{\Phi})$ is given by

$$\epsilon(t,\overline{\Phi}) = \frac{(1 - \Phi_1 q^{-1} - \Phi_2 q^{-2})y(t)}{1 - \mathcal{O}_1 q^{-1}} \qquad (9)$$

where q is a shift operator. From (8) and (9) the following equations are obtained

$$\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_1} = \frac{-q^{-1}}{1 - \mathcal{O}_1 q^{-1}} y(t) \qquad (10)$$

$$\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_2} = \frac{-q^{-2}}{1 - \mathcal{O}_1 q^{-1}} y(t) \qquad (11)$$

Since (2) is assumed to give a true description of the system in (1) the following equations are obtained

$$\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_1} = \frac{-q^{-1}}{1-\Phi_1 q^{-1}-\Phi_2 q^{-2}}e(t) \tag{12}$$

$$\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_2} = \frac{-q^{-2}}{1-\Phi_1 q^{-1}-\Phi_2 q^{-2}}e(t) \tag{13}$$

It is now possible to calculate $\overline{\overline{J}}$ from (7), (12) and (13) in the following way. The inverse of the Fisher information matrix is

$$\overline{\overline{J}} = \frac{N}{\lambda_\mathcal{E}}\begin{bmatrix} E[\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_1}]^2 & E[\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_1}\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_2}] \\ E[\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_1}\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_2}] & E[\frac{\partial\epsilon(t,\overline{\Phi})}{\partial\Phi_2}]^2 \end{bmatrix} \tag{14}$$

The elements in the information matrix can be written in the frequency domain by

$$J_{11} = \frac{\lambda_\mathcal{E}}{2\pi}\int_{-\pi}^{\pi}\frac{e^{-i\omega}}{1-\Phi_1 e^{-i\omega}-\Phi_2 e^{-2i\omega}}\frac{e^{i\omega}}{1-\Phi_1 e^{i\omega}-\Phi_2 e^{2i\omega}}d\omega \tag{15}$$

$$J_{22} = \frac{\lambda_\mathcal{E}}{2\pi}\int_{-\pi}^{\pi}\frac{e^{-2i\omega}}{1-\Phi_1 e^{-i\omega}-\Phi_2 e^{-2i\omega}}\frac{e^{2i\omega}}{1-\Phi_1 e^{i\omega}-\Phi_2 e^{2i\omega}}d\omega \tag{16}$$

$$J_{12} = \frac{\lambda_\mathcal{E}}{2\pi}\int_{-\pi}^{\pi}\frac{e^{-i\omega}}{1-\Phi_1 e^{-i\omega}-\Phi_2 e^{-2i\omega}}\frac{e^{2i\omega}}{1-\Phi_1 e^{i\omega}-\Phi_2 e^{2i\omega}}d\omega \tag{17}$$

It is seen from (14), (15), (16) and (17) that the information matrix does not depend on the variance of the noise process.

Using that the complex number z is given by

$$z = e^{i\omega} \tag{18}$$

and

$$dz = ie^{i\omega}d\omega = izd\omega \tag{19}$$

(15) can be written

$$J_{11} = \frac{\lambda_\mathcal{E}}{2\pi i}\oint\frac{z^{-1}}{1-\Phi_1 z^{-1}-\Phi_2 z^{-2}}\frac{z}{1-\Phi_1 z^{1}-\Phi_2 z^{2}}\frac{dz}{z} \tag{20}$$

$\oint$ is a complex integration around the unit circle $|z| = 1$, counterclockwise. The complex integration around the unit circle can now be evaluated by using residue calculus, see e.g. Kreyzig [10]

$$\oint f(z)dz = 2\pi i\sum_{j=1}^{k} Resf(z_j) \tag{21}$$

where $z_j = 1, 2, .., k$ are the singular points inside the unit circle. The singular points in (21) are

$$z_1 = \frac{\Phi_1}{2} + \frac{1}{2}\sqrt{\Phi_1^2 + 4\Phi_2} \tag{22}$$

$$z_2 = \frac{\Phi_1}{2} - \frac{1}{2}\sqrt{\Phi_1^2 + 4\Phi_2} \tag{23}$$

$$z_3 = \frac{\Phi_1}{2\Phi_2} + \frac{1}{2\Phi_2}\sqrt{\Phi_1^2 + 4\Phi_2} \tag{24}$$

$$z_4 = \frac{\Phi_1}{2\Phi_2} - \frac{1}{2\Phi_2}\sqrt{\Phi_1^2 + 4\Phi_2} \tag{25}$$

From the above it is seen that

$$\Phi_1^2 + 4\Phi_2 \leq 0 \tag{26}$$

given that f and Δt are both positive and real as long as ζ stays positive and real. This implies that the complex roots are complex conjugate pairs and

$$|z_1| = |z_2| = e^{-\zeta\omega\Delta t} < 1; \Delta t > 0 \tag{27}$$

$$|z_3| = |z_4| = e^{\zeta\omega\Delta t} > 1; \Delta t > 0 \tag{28}$$

This means that the roots z_1 and z_2 are singular points within the unit circle while z_3 and z_4 are singular points outside the unit circle, i.e. it is only the singular points z_1 and z_2 which shall be taken into account when the complex integral in (20) is calculated.

The calculation of the integral can be made by using (see e.g. Kreyzig [10])

$$Resf(z_j) = Res\frac{p(z_j)}{q(z_j)} = \frac{p(z_j)}{q'(z_j)} \tag{29}$$

where $'$ denotes a derivative of $q(z)$ with respect to z.

Above it is explained how one element in the information matrix can be calculated. The other elements can be calculated in the same way.

When the elements of the information matrix are calculated the parameter covariance matrix $\overline{\overline{C}}_{\hat{\overline{\theta}}_N}$ of estimates of the parameter vector $\hat{\overline{\theta}}_N$ can be expressed in the following way

$$\overline{\overline{C}}_{\hat{\overline{\theta}}_N} = \overline{\overline{A}}\,\overline{\overline{J}}^{-1}\overline{\overline{A}}^T \tag{30}$$

where the transformation matrix $\overline{\overline{A}}$ is given by

$$\overline{\overline{A}} = \begin{bmatrix} \frac{\partial f}{\partial\Phi_1} & \frac{\partial f}{\partial\Phi_2} \\ \frac{\partial\zeta}{\partial\Phi_1} & \frac{\partial\zeta}{\partial\Phi_2} \end{bmatrix} \tag{31}$$

Since the connection between the AR-parameters and the parameters $\overline{\theta} = [f, \zeta]^T$ is non-linear the transformation matrix $\overline{\overline{A}}$ cannot be directly obtained. Instead of the following relation is used

$$\overline{\overline{B}}\,\overline{\overline{A}} = \overline{\overline{I}} \tag{32}$$

where $\overline{\overline{I}}$ is the identity matrix. Then

$$\overline{\overline{A}} = \overline{\overline{B}}^{-1} \tag{33}$$

The matrix $\overline{\overline{B}}$ is given by

$$\overline{\overline{B}} = \begin{bmatrix} \frac{\partial\Phi_1}{\partial f} & \frac{\partial\Phi_1}{\partial\zeta} \\ \frac{\partial\Phi_2}{\partial f} & \frac{\partial\Phi_2}{\partial\zeta} \end{bmatrix} \tag{34}$$

The above estimation of $\overline{\overline{A}}$ will only be accurate if the function is sufficiently smooth since it corresponds to a linear approximation of the function describing the inverse transformation from AR- parameters to the parameters $\overline{\theta}$.

The covariance matrix of the parameter estimates $\hat{\overline{\theta}}_N$ is now expressed as a function of the sampling interval Δt and number of samples N. This analytical connection makes it easy to consider the problem whether there is an optimal sampling rate or not.

3. RESULTS

A simulation study is performed to investigate the applicability of the analytical solution.

The most accurate way to perform simulations of a SDOF system formulated in continuous time is to transform the system model into discrete time space which can be done by using the ARMA(2,1) model described above. By using this model the response is simulated for a SDOF system. The parameters are estimated by an ARMA(2,1) model and the expected values $\mu_{\hat{\overline{\theta}}_N}$ of the parameter estimates $\hat{\overline{\theta}}_N$ and the covariance matrix $\overline{\overline{C}}_{\hat{\overline{\theta}}_N}$ are estimated by the sample averages

$$\mu_{\hat{\overline{\theta}}_N} = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} \hat{\overline{\theta}}_N^{(i)} \tag{35}$$

$$\overline{\overline{C}}_{\hat{\overline{\theta}}_N} = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} (\hat{\overline{\theta}}_N^{(i)} - \mu_{\hat{\overline{\theta}}_N})(\hat{\overline{\theta}}_N^{(i)} - \mu_{\hat{\overline{\theta}}_N})^T \tag{36}$$

where N_{sim} is the number of simulations. The simulation study is performed using the MATLAB software package on a VAX 8700 computer. A description of the MATLAB software package can be found in PC-MATLAB [11].

In table 1 the parameter estimation uncertainty based on simulations (SIM) of the response is shown with values obtained by the analytical (ANA) solution for a SDOF system. δ_f and δ_ζ are the coefficient of variation of the eigenfrequency and the damping, respectively. $\rho_{f,\zeta}$ is the correlation coefficient between the eigenfrequency and the damping.

The simulation study is performed with the following data:

f=1 Hz., ζ=0.005 and 0.02, $\Delta t = 0.3$, N_{sim}=100, N=8000

	ANA	SIM	ANA	SIM
ζ	0.005	0.005	0.02	0.02
δ_f	0.00058	0.00063	0.00125	0.00130
δ_ζ	0.1163	0.1253	0.0598	0.0599
$\rho_{f,\zeta}$	0.0032	0.1665	0.0142	0.0619

Table 1: Parameter estimation uncertainty based on simulations (SIM) of the response is shown with values obtained by the analytical (ANA) solution for a SDOF system.

Comparing the analytical and the simulated results for the coefficient of variation it is seen that the analytical results predict rather well what is to be expected in the practical simulation. It is seen that they deviate less than 10 per cent. On the other hand, it is seen that the prediction of the analytical correlation coefficient estimates is fairly uncertain. Thus, it can be concluded that the theoretical values obtained for the parameter uncertainty give a good indication of the parameter uncertainty obtained from practical simulations. Further, it is seen that the coefficient of variation of the eigenfrequency is proportional with the damping ratio and that the coefficient of variation of the damping ratio is inverse proportional with the damping ratio.

In the following the analytical solution will be used to investigate the problem whether there is an optimal sampling interval or not. The SDOF system is modelled by the eigenfrequency and damping ratio mentioned above. The number of samples is N=8000.

In figure 1 and figure 2, respectively, the coefficient of variation of the eigenfrequency and the damping ratio are shown for different sampling intervals Δt. The sampling interval is varied from $0 \rightarrow 0.5$ corresponding to the Nyquist frequency being equal to the resonance frequency of the system. Considering the coefficients of variations as functions of the sampling interval it is seen that they have minima for certain values of Δt. It is also seen that the functions are flat near the minima. This causes difficulties in the precise choice of the optimal sampling interval on the one hand, but it also means that some imperfections in the optimally chosen sampling interval result in relatively small increase in error. However, it is seen that the coefficients of variation increase rapidly when the sampling interval Δt increases from the optimal value giving the minima. Thus, it is far worse to use a too large sampling interval than a too small one when the number of samples N is constant.

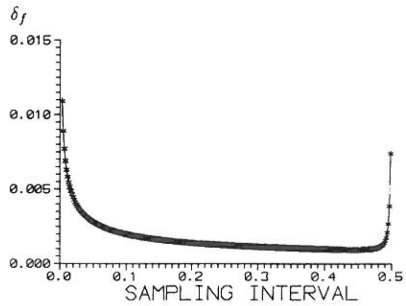

Figure 1: The coefficient of variation δ_f shown as functions of Δt. (N = 8000)

The results shown in figure 1 and 2 correspond to results given in Åström [12] where a first order system is analysed. The result that an optimal sampling interval exists when N is fixed has also been obtained in Jensen et al. [13] by a simulation study of a second order system excited by white noise.

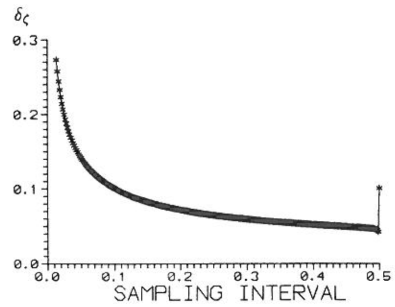

Figure 2: The coefficient of variation δ_ζ shown as functions of Δt. (N = 8000)

In figure 3 and figure 4 the coefficient of variation of the eigenfrequency and the damping ratio is shown for different sampling intervals Δt, where the total experiment length $T_f = N\Delta t$ is kept constant and the number of samples N is varied. Here T_f=1000 sec. For this case it is seen that there is no optimal sampling interval. The coefficients of variation are monotonically increasing when the sampling interval increases.

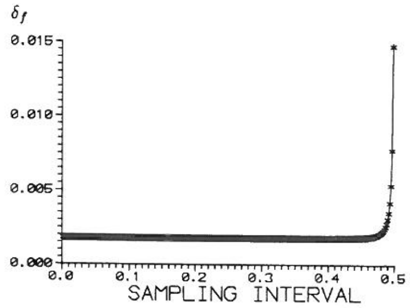

Figure 3: The coefficient of variation δ_f shown as functions of Δt. (T_f=1000 sec.)

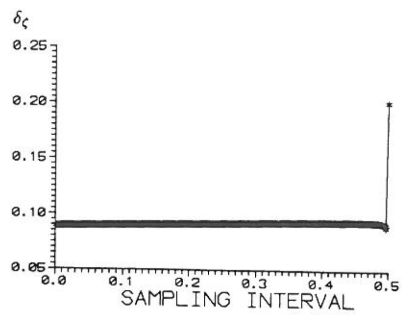

Figure 4: The coefficient of variation and δ_ζ shown as functions of Δt. (T_f=1000 sec.)

In figure 5 the determinant of the parameter covariance matrix is shown for different sampling intervals Δt where N is constant. In figure 6 the determinant of the parameter covariance matrix is shown for different sampling intervals Δt where T_f is constant. It is seen that an optimal sampling interval exists for N constant and not for T_f constant.

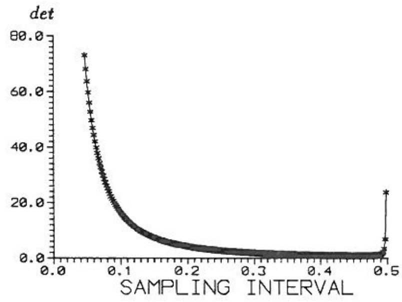

Figure 5: The determinant of the parameter covariance matrix shown as a function of Δt. (N=8000)

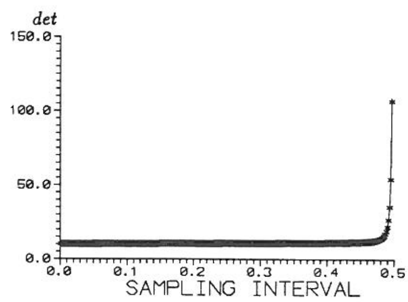

Figure 6: The determinant of the parameter covariance matrix shown as a function of Δt. (T_f = 1000$sec.$)

Table 2 shows the optimal sampling intervals Δt^{opt} corresponding to the figures shown above. Further the optimal sampling interval obtained by using a trace (tr) scalar measure of the parameter covariance is also shown. The third and fourth column show the optimal sampling interval corresponding to a minimum of the coefficient of variation of the eigenfrequency and damping ratio, respectively.

	det	tr	δf	$\delta\zeta$
Δt^{opt}	0.462	0.456	0.445	0.499

Table 2: Optimal sampling interval for different design criteria

Table 2 shows that the choice of optimal sampling interval depends on the design criteria. However, it is seen that the optimal choice of the sampling interval only deviates a little for the different criteria.

In order to investigate the sensitivity of the optimal sampling interval to a variation of the damping ratio and the eigenfrequency f for a fixed number of samples N the dimensionless quantity $\Delta t^{opt} f$ is calculated for different values of ζ, see figure 7. The optimal sampling intervals are determined by the determinant of the parameter covariance matrix.

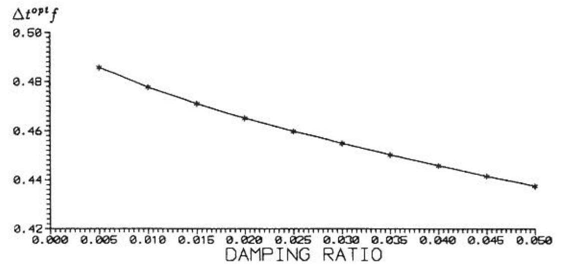

Figure 7: The dimensionless quantity $\Delta t^{opt} f$ shown as function of Δt.

From the results used in figure 7 the dimensionless quantity $\Delta t^{opt} f$ is fitted by a linear expression giving

$$\Delta t^{opt} = (0.4876 - 1.045\zeta) f^{-1} \tag{37}$$

It is seen that the optimal sampling interval is proportional to ζ^{-1} and f^{-1}. By using (37) it is possible to estimate an optimal sampling interval based on a priori information about the modal parameters. However the correct values of the modal parameters cannot be known a priori. This means that the optimal sampling interval has to be determined by an iterative procedure where one first has to choose an initial guess of the sampling interval. Next an estimate of the modal parameters can be determined giving a new estimate of the optimal sampling interval

Until now the optimal choice of the sampling interval has been investigated. It may be noticed that when analytical calculations like those in the above example are too laborious to carry out or other model structures are necessary, simulations can be used to estimate the best sampling rate. Such simulations may be time-consuming. However, the choice of the experiment length has not been considered. In practice, the experiment length is often limited due to stationarity requirements or purely to practical considerations. The restriction on the number of data is frequently met in practice due to the cost of data acquisition or computer storage restrictions. If the data storage permits and if cheap data acquisition is dealt with sampling at a high sampling rate can be made. This data may then be filtered and desampled to the desired frequency before performing the final data analysis. Thus one is dealing with

- a sampling interval for data acquisition,
- a sampling interval for final parameter estimation.

However, it is seen from figure 8 that when the experiment length, (number of sample points) has reached a given magnitude only limited improvement can be obtained by increasing N. One way to determine the optimal choice of N could be a cost-benefit analysis, see Kirkegaard [7].

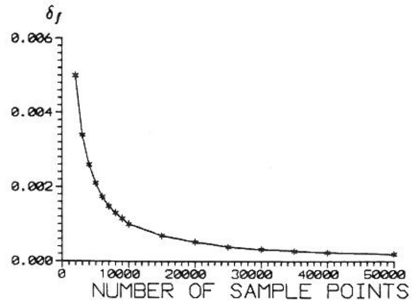

Figure 8: The influence of number of sample points on the coefficient of variation of the eigenfrequency

4. CONCLUSIONS

In this paper optimal selection of the sampling interval for estimation of the modal parameters by an ARMA-model for a white noise loaded structure modelled as a single degree-of-freedom linear mechanical system is considered. The conclusions of the paper can be stated as follows:

- An analytical solution for an optimal uniform sampling interval, which is optimal in a Fisherian sense, is given. It is seen that the analytical results predict rather well what is to be expected in the practical simulation.
- If the experimental length T_f is fixed it may be useful to sample the record at a high sampling rate, since more measurements from the system are then collected. No optimal sampling interval exists.
- If the total number of sample points N is fixed an optimal sampling interval exists. Then it is far worse to use a too large sampling interval than a too small one since the information losses increase rapidly when the sampling interval increases from the optimal value.
- In principle, the choice of optimal experimental length is a cost-benefit problem where the value of achieving extra information has to be balanced against the cost of obtaining the information.

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Department of Building Technology and Structural Engineering
The University of Aalborg, Sohngaardsholmsvej 57, DK 9000 Aalborg
Telephone: 45 98 15 85 22 Telefax: 45 98 14 82 43